A Model Order Reduction Approach for Finite Element Method in Time Domain Simulations in Microwave Circuits

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***Abstract*—Time domain simulations of electromagnetic problems are intrinsically interesting for engineering purposes, not only from the physical point of view but also from the computational effort. These approaches rely on a marching on time schemes where the field solution can be obtained from previous time steps in an increasing manner. However, each new sample requires computations with a small complexity (typically matrix-vector multiplications) but within a large approximation dimension, which clearly deteriorates the simulation time.**

A model order reduction approach for finite element method in time domain (FEMTD) simulations for microwave devices using time as a parameter is proposed in this work. This methodology shows the possibility to solve time evolution problems in electromagnetics requiring a small computational effort. Indeed, the system matrices involved in the model order reduction technique are pretty small. This is in contrast to the large dimension matrices arisen in traditional FDTD and FEMTD approaches.

Several microwave circuits, such as an electromagnetic band gap structure and dielectric resonator filter, will show the capabilities and possibilities of this new approach for time domain simulations in electromagnetics.

***Index Terms*—Computational electromagnetics (CEM), computational prototyping, finite element methods, model order reduction, microwave circuits and antennas, numerical techniques, simulation and optimization.**

I. Introduction

Computational electromagnetics is a critical tool to assist microwave engineers. Any speed up obtained by any simulation tool in CEM gives rise to better electromagnetic designs. As a result, a huge effort is placed in industry and academia to push simulation tools forward.

An important approach in CEM to reduce computational resources are the so-called model order reduction approaches [1]–[4]. A reduced-order model (ROM) consists of replacing a rather complex physical model by a much simpler mathematical one but still maintains certain physical aspects of the original model along a parameter set.

Along the same path, time domain simulations take advantage from special time integrator schemes which are capable to provide wideband responses for microwave systems by using low complexity algorithms, which ideally only involve matrix-vector multiplications. As a result, these approaches provide good bandwidth-computation time ratios. Finite difference in time domain (FDTD), finite element method in time domain (FEMTD), discontinuous galerkin in time domain (DGTD), to name a few, are popular numerical approaches [5]–[12].

We propose a model order reduction approach for FEMTD simulations. This approach can straightforwardly be extended to other numerical technique for time domain simulations. In fact, [13] uses a dynamic mode decomposition (DMD) approach for FDTD simulations in cavity problems.

This work is structured as follows. Section II presents the discretization setting for FEM formulations in time domain. Section III introduces the model order reduction approach. Section IV details the possibilities of the proposed approach with difference microwave circuits. Finally, in Section V, we comment on the conclusions.

II. Problem Statement

Time-dependent Maxwell's equations in the analysis domain $\Omega \subset \mathbb{R}^3$ can be written in a classical weak formulation over an appropriate admissible function space $\mathcal{H}$, viz.

$$\begin{aligned} &\text{find } \mathbf{E} \in \mathcal{H} \text{ such that} \\ &\quad a(\mathbf{E}, \mathbf{v}) = f(\mathbf{v}) \ \forall \mathbf{v} \in \mathcal{H}. \end{aligned} \tag{1}$$

The bilinear form being

$$a(\mathbf{E}, \mathbf{v}) = \int_{\Omega} \left(\frac{1}{\mu} \nabla \times \mathbf{E} \cdot \nabla \times \mathbf{v} + \sigma \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{v} + \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \cdot \mathbf{v} \right) dx, \tag{2}$$

and the linear form

$$f(\mathbf{v}) = -\int_{\Gamma} \frac{\partial \mathbf{J}}{\partial t} \cdot \mathbf{v} \; ds, \tag{3}$$

where μ is the magnetic permeability, ε is the dielectric permittivity, σ denotes the conductivity and $\mathbf{J}$ is the excitation current on surface $\Gamma \subset \partial\Omega$. Here, the admissible space $\mathcal{H}$ is a subspace in the Hilbert space $H(curl, \Omega)$ since appropriate boundary condition should be taken into account, namely, PEC boundary conditions,

$$\mathcal{H} = \{ \mathbf{u} \in H(curl, \Omega) \mid \mathbf{n} \times \mathbf{u} = \mathbf{0} \text{ on } \Gamma_{\text{PEC}} \}. \tag{4}$$

The variational problem (1) is typically solved by means of the finite element method (FEM), which provides a large dimension (but finite) approximation space $\mathcal{H}^h$ to the (infinite dimension) admissible function space $\mathcal{H}$. This approach determines the geometry space discretization of the electric field $\mathbf{E}(t)$ in the analysis domain $\Omega \subset \mathbb{R}^3$, but the time evolution problem still remains. After this space discretization, the time evolution in the electric field in (1) turns into an ODE problem, thus,

$$\mathbf{SE}(t) + \mathbf{U}\frac{d\mathbf{E}(t)}{dt} + \mathbf{T}\frac{d^2\mathbf{E}(t)}{dt^2} = \mathbf{b}\frac{d\mathbf{i}(t)}{dt}, \quad (5)$$

which can be solved by any time integrator scheme of your choice. In this work, we use the Newmark-β method [14], [15]. Matrices $\mathbf{S}$, $\mathbf{U}$ and $\mathbf{T}$, are the sparse FEM matrices of dimension N, where N is the dimension of the finite element approximation space $\mathcal{H}^h$ ($N \gg 1$). Matrix $\mathbf{b}$ takes into account the excitation on the ports and $\mathbf{i}(t)$ is the input current.

III. Model Order Reduction

The evolution of the electric field $\mathbf{E}(t)$ can be obtained by any time integrator in the FEM approach detailed in Section II. However, this time integration will rely not only on marching on time evaluations of the electric field (this is a must in any time domain scheme), but also on large dimension matrix operations (which involves the FEM approximation dimension space N, $N \gg 1$). As a result, this may turn into a rather time-consuming process. To significantly speed up simulation time in time integrator schemes, we propose to use a model order reduction approach.

A reduced-order model replaces a rather complex physical model by a much simpler mathematical one that still maintains certain physical aspects of the original model within a parameter set. The computational complexity of the ROM should be insignificant in contrast to the high computational cost of the original full order model. In our approach, we will consider time t as a parameter.

Let us consider the electric field solution manifold we are interested in taking time as parameter, namely,

$$\mathcal{M} = \{\mathbf{E}(t) \subset \mathcal{H}, t \in [t_0, t_1] \subset \mathbb{R}\}. \quad (6)$$

The electric field $\mathbf{E}(t)$ does not arbitrarily evolve in the infinite dimension admissible function space $\mathcal{H}$, on the contrary, it follows a very specific trajectory within $\mathcal{H}$. This gives rise to the solution manifold $\mathcal{M}$, and we claim this solution manifold can be approximated by a quite small dimension solution space $\mathcal{H}_M$ of dimension M, the reduced-basis space. This dimension is very small in comparison with the dimension of the finite element approximation space ($M \ll N$). Putting everything together, $\mathcal{H}_M \subset \mathcal{M} \subset \mathcal{H}^h \subset \mathcal{H}$.

Instead of using the finite element approximation space $\mathcal{H}^h$ to solve (1), we propose to solve the same problem but with the reduced-basis approximation space $\mathcal{H}_M$. Following a similar procedure, we obtain the same ODE that in (5) but, this time, matrices $\mathbf{S}, \mathbf{U}, \mathbf{T}$ and $\mathbf{b}$ are matrices of dimension M ($M \ll N$). As a result, we can solve the time evolution problem with ease using the reduced-basis approximation space.

A question still remains: how can we find this reduced-basis space $\mathcal{H}_M$? No miracle though, we need to rely on the FEMTD time integrator to actually find out the reduced-basis space. We propose to use the first m samples in the FEMTD simulations for the electric field, so-called snapshots, namely,

$$\mathbf{E}_0, \mathbf{E}_1, \ldots, \mathbf{E}_{m-1}, \quad (7)$$

where $\mathbf{E}_n = \mathbf{E}(t0 + n\Delta t)$, and Δt is the time step used in the time integrator. Now, we carry out the SVD of all these snapshots arranged into a matrix and use the first M left singular vectors as a basis for the reduced-basis space $\mathcal{H}_M$ ($M < m$). This clears up all linear dependency in the field solution snapshots and a suitable reduced-basis approximation space arises.

In the following section, we will provide several examples to show the capabilities of this model order reduction approach.

IV. Numerical Results

In this section, we apply the proposed model order reduction approach to analyze the responses from several microwave circuits, namely, an electromagnetic band gap (EBG) surface inside a parallel-plate waveguide and a dielectric resonator filter. The in-house C++ code for FEM simulations uses a second-order first family of Nédélec's elements, on second-order tetrahedral meshes provided by `Gmsh` [16]. All computations were carried out on a workstation with two 3.00-GHz Intel Xeon E5-2687W v4 processors and 512-GB RAM.

A. Mushroom-type Electromagnetic Band Gap Structure

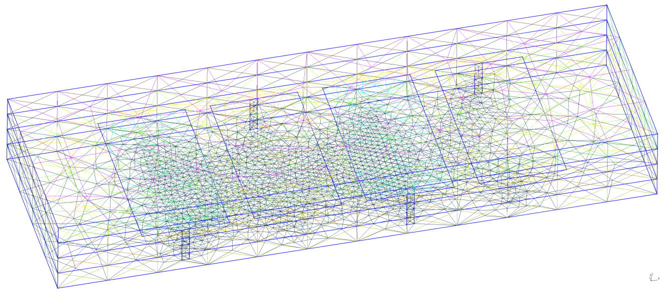
A mushroom-type EBG surface inside a dielectric-filled parallel-plate waveguide proposed in [17] is studied. The geometry and mesh of this EBG is shown in Fig. 1. A detail from the mushroom-type metallic stop-band resonators is depicted in Fig. 1b. Transmission zeroes should be expected in the frequency response to build a stopband behavior in the parallel-plate waveguide. An FEM discretization with 95,058 degrees of freedom is used.

The 1-5 GHz band is considered for wideband analysis. As a result, a Gaussian pulse with center frequency 3 GHz and 0.25 ns pulse width is taken into account as excitation. The FEMTD solver is run for 2 ns with a 5 ps time step (Δt) providing 400 snapshots. We carry out the SVD for the snapshots and take the first 25 left singular vectors. These gives rise to our reduced-basis space, and now we solve the time evolution problem in (5) with matrices of dimension 25, which is straightforward to carry out. The reduced-order model is run (which implies matrix operations of dimension 25) for 120 ns with a 5 ps time step. The simulation results are compared with FEM in the frequency domain analysis in Fig. 2. Good agreement is found between approaches.

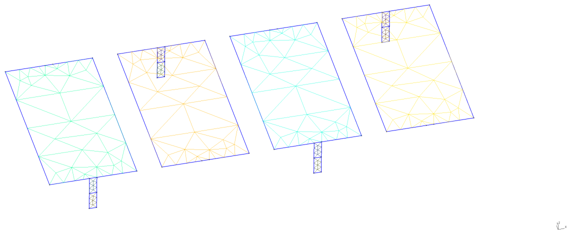
B. Coax-fed Dielectric Resonator Filter

Next, we consider a coax-fed dielectric resonator filter originally proposed in [18]. The geometry of this filter is shown in Fig. 3. This filter consists of two cylindrical dielectric resonators, each having a concentric hole. An FEM discretization with 56,652 degrees of freedom is used.

The 1-5 GHz band is considered for wideband analysis. Once again, a Gaussian pulse with center frequency 3 GHz and 0.25 ns pulse width is taken into account as excitation. The FEMTD solver is run for 2 ns with a 5 ps time step (Δt) providing 400 snapshots. We carry out the SVD for the snapshots and take the first 25 left singular vectors. These gives rise to our reduced-basis space, and now we solve the time evolution problem in (5) with matrices of dimension



(a)



(b)

Fig. 1. Geometry and mesh of the parallel-plate waveguide with a mushroom-type EBG surface proposed in [17]. (a) Dielectric-filled parallel-plate waveguide overalll view. (b) Detail of the mushroom-type metallic resonators.

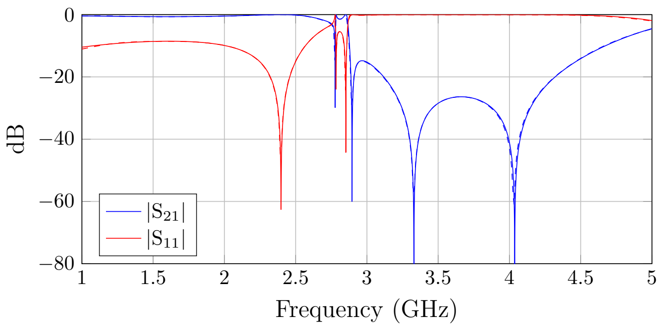


Fig. 2. Parallel-plate waveguide with EBG scattering parameter response. (–) FEM in frequency domain analysis. (- -) ROM analysis in FEMTD.

25. The reduced-order model is run for 120 ns with a 5 ps time step. This model order reduction approach requires only matrix operations of dimension 25. The simulation results are compared with FEM in the frequency domain analysis in Fig. 4. Reasonable agreement is met between approaches.

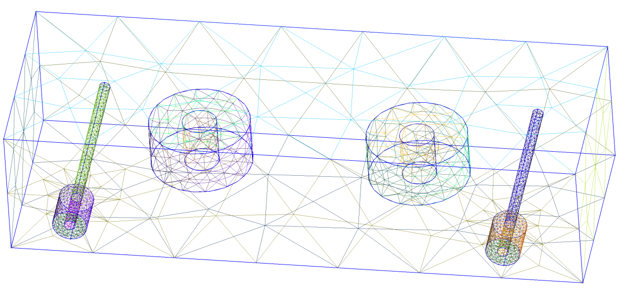


Fig. 3. Geometry of the coax-fed dielectric resonator filter from [18].

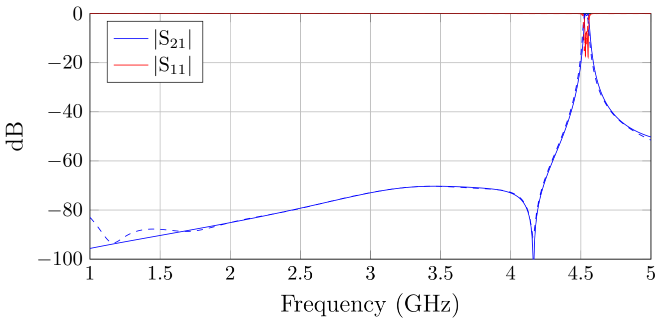


Fig. 4. Coax-fed dielectric resonator filter scattering parameter response. (–) FEM in frequency domain analysis. (- -) ROM analysis in FEMTD.

V. Conclusions

A model order reduction approach for FEMTD simulations for microwave devices using time as a parameter have been proposed. This methodology has shown the possibility to solve time evolution problems in electromagnetics by using a small computational effort since the system matrices involved in the ROM formulation are pretty small. This is in contrast to the large dimension matrices arisen in traditional FDTD and FEMTD approaches.

A mushroom-type EBG structure inside a parallel-plate waveguide and a dielectric resonator filter have been used to show the capabilities and accuracy of the proposed model order reduction strategy for time domain simulations in computational electromagnetics.

Acknowledgements

This work has been developed in the frame of the activities of the Project PULSE, funded by the European Innovation Council under the EIC Pathfinder Open 2022 program (protocol number 101099313). Project website is: https://www.pulse-pathfinder.eu/.

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